

The New WBF IMP to VP Scales

Technical Report of WBF Scoring Panel

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1 Introduction

This document presents the theory and algorithms for producing the new WBF conversion tables from:

1. IMPs to continuous VPs; and
2. IMPs to discrete VPs.

The continuous scale gives a unique Victory Point (VP) to two decimal places for each integer IMP margin. The discrete scale, similar to existing WBF scales, gives a range of IMPs for each integer VP score. VP scores for both scales range from 0 VPs for a maximum loss, to 20 VPs for a maximum win. A drawn match results in a VP score of 10–10 to each team. If team A wins V Victory Points, then its opponent, team B, wins $(20 - V)$ Victory Points.

These scales differ in two significant ways when compared to the old scales.

1. The continuous scale removes the phenomenon of ‘cussing’, where one extra IMP gained could have led to a full extra VP. In the continuous scale, each extra IMP leads to a non-increasing fractional gain in VPs.
2. The new discrete scale removes anomalies in the old scales. In particular, the IMP range is non-decreasing for each subsequent integer VP score. The old WBF scales violated this concavity rule, particularly for small IMP margins.

Both new scales (continuous and discrete) ensure that IMPs earned in a close match are worth more than subsequent IMPs earned in a run-away match. The new continuous scale is identical, except for some minor corrections, to the USBF scale devised by Henry Bethe, that has been successfully used in the round robin phases of North American team trials for some years.

2 Theoretical Background

Both continuous and discrete scales are based on an exponential mapping of IMPs to VPs described by the following formula

$$V_{\text{map}}(I; V_0, X) = V_0 + V_0 \left(\frac{1 - R^{I/X}}{1 - R} \right); \quad I \geq 0 \quad (1)$$

$$R = \tau^3; \quad \tau = \frac{1}{2}(\sqrt{5} - 1) \quad (2)$$

Remarks:

1. I is the IMP margin (a non-negative integer); $V_0 = 10$ is the VP score for a drawn match (*i.e.* $V_{\text{map}} = V_0$ when $I = 0$); X is the blitz (maximum win) IMP margin determined by the formula:

$$X = 15\sqrt{N} \quad (3)$$

where N is the number of boards played in the current match. Note that X will in general not be an integer. The two parameters (V_0, X) allow for possible variations in the choice of VP range and blitz point respectively.

2. Formula (3) comes from a study of some 200,000 individual board records from BBO matches. These data indicated (when assuming roughly equally matched teams) a symmetric distribution with zero mean and standard deviation equal to 7.51 IMPs per board. The distribution is highly non-normal with secondary peaks at ± 6 and ± 11 IMPs, due to non-vulnerable game and vulnerable game and slam swings. If results for multiple boards are independent we can safely use the Central Limit Theorem to infer that the standard deviation for N boards will be $7.51\sqrt{N}$ IMPs. Thus the blitz point $X = 15\sqrt{N}$ is seen to be very close to two standard deviations from the mean.
3. Henry Bethe also inferred from World Championship and other round robin teams matches that the median IMP margin is approximately $5\sqrt{N}$ IMPs. If we assign this IMP margin to a VP score of 15 VPs (assuming a 0–20 VP scale) then about half the winning results will lie in the range 10–15 VPs and half in the range 15–20 VPs. Commensurately, if V_0 is the VP score for a drawn match, $2V_0$ is the maximum VP score and the median condition above becomes

$$V\left(\frac{1}{3}X\right) = \frac{3}{2}V_0 \quad (4)$$

Bethe also found from the same data, that between 5% and 8% of all such encounters had margins that exceeded $15\sqrt{N}$ IMPs, a result which is consistent with the roughly two standard deviations blitz point noted in item 2. above.

4. The parameter τ is the so-called *golden mean* and R is its cube. Thus τ and R are just numerical factors and to six decimal places are given respectively by

$$\tau = \frac{1}{2}(\sqrt{5} - 1) = 0.618034; \quad R = \sqrt{5} - 2 = 0.236068 \quad (5)$$

How these factors come into the scale is a direct consequence of the equations described in Item 6. below.

5. The mapping described by Eq(1) is a monotonic increasing and concave exponential function of the IMP margin I . That is, its first and second derivatives (with respect to I) are respectively positive and negative. The concavity condition $\frac{d^2V}{dI^2} < 0$ is regarded as an important property of the IMP to VP conversion and ensures that VPs are more sensitive to smaller IMP margins than to larger ones. Note that linear conversion scales, to which the old WBF scales approximate, have constant sensitivity at all IMP margins below the blitz point. Of course, above the blitz point, the sensitivity is zero as each new IMP gained has no effect on the maximum VP score ($2V_0 = 20$) already attained.
6. The actual motivation for the exponential scale is discussed here and in the next two items. Non-mathematicians can safely skip these technical issues.

Let x be a hypothetical IMP margin (not necessarily an integer) and $V(x)$ be its corresponding VP score. We define the sensitivity $S(x)$ of the scale by the first derivative and the concavity $C(x)$ by the negative of the second derivative (which means $C(x) > 0$). That is

$$S(x) = \frac{dV}{dx} \quad \text{and} \quad C(x) = -\frac{d^2V}{dx^2} \quad (6)$$

Then the exponential scale defined by Eq(1) is the unique solution of the following sensitivity equation and constraints (here A and B are

positive constants):

$$\left. \begin{aligned} S(x) &= A - B V(x) \\ V(0) &= V_0 \\ V(X) &= 2V_0 \\ V(\frac{1}{3}X) &= \frac{3}{2}V_0 \end{aligned} \right\} \quad (7)$$

The sensitivity is modelled as a linear decreasing function of the VP score. The three constraints respectively refer to the draw point, the blitz point and the median point – all mentioned previously.

7. Other models for the sensitivity are of course possible. For example:

Sensitivity Model	Conversion Scale
$S(x)$	$V(x)$
$A - Bx$	Quadratic
$A - BV(x)$	Exponential
$\frac{1}{A+Bx}$	Logarithmic
$\frac{1}{A+BV(x)}$	Square-root

The quadratic and square-root scales tend to give extreme IMP to VP conversions, sensitivities and concavities while the exponential and logarithmic scales are more moderate. The WBF Scoring Panel considered all four models before adopting the exponential scale.

8. As the IMP margin is always an integer and because we round the VPs to two decimal points, the new continuous scale is really only a pseudo-continuous scale. We are content to refer to this scale as a continuous scale mainly to distinguish it from the discrete scale considered later in this report.

The continuous WBF scale requires only the function defined by Eq(1). However, the discrete WBF scale requires its inverse. That is, given the winner's VP score V , the IMP mapping I_{map} that generates this VP value. This is the logarithmic function

$$I_{\text{map}}(V; V_0, X) = X \left[\frac{\log\{1 - (1 - R)(V/V_0 - 1)\}}{\log(R)} \right] \quad (8)$$

provided $V_0 \leq V \leq 2V_0$. Note that both Eqs(1) and (8) apply only to the winner's VPs and IMPs respectively. Naturally, if I is the IMP margin, then the winner's and loser's actual IMP scores must have been $+I$ and $-I$

respectively.

The next two sections describe in detail the algorithms that produce the continuous scales (to two decimal points) and the discrete scales for any prescribed board numbers in a given match.

3 The Continuous VP Scale

The IMP margin is any non-negative integer, whereas the continuous VP scores are presented to two decimal places. Each IMP margin leads to a unique VP score. The VP range is from zero VPs (maximum loss) to 20 VPs (maximum win). A draw corresponding to zero IMP margin results in a 10–10 VP score. The algorithm below determines the winners score of V_{win} (VPs) say. Then the loser will receive $V_{lose} = (20 - V_{win})$ (VPs).

The pseudo-codes below should permit programmers to convert the codes to their preferred programming language.

The IMP-to-Continuous-VP Algorithm

1. For IMP margin $i = 0$ to $\text{ceil}(X)$ in steps of 1–IMP compute the VP table

$$V_i = \min \left[\frac{\text{round}(100 * V_{\text{map}}(i, V_0, X))}{100}, 2V_0 \right] \quad (9)$$

i.e. for $i = 0, 1, 2, \dots, \text{ceil}(X)$ and $X = \sqrt{N}$ with $N = \text{no. of boards}$.

Remarks:

$\text{ceil}(X)$ is the ceiling-function of X defined to be the smallest integer exceeding X (*i.e.* round towards infinity). The function $\text{round}(Y)$ takes any real number Y and rounds it to the nearest integer value. A value ending in exactly point-five is rounded up to the next integer. The function $V_{\text{map}}(i, V_0, X)$ is given by Eq(1). This formula essentially gives the VP's to two decimal places, while cutting off the maximum VP value at $2V_0 = 20$ VPs.

2. The rounding in step–1 of the algorithm sometimes produces a violation of the Concavity Rule. Step–2 is designed to correct for all such violations. Generally the corrections to V amount to only 0.01 of a VP. The concavity rule can be represented as follows. If (V_{i-1}, V_i, V_{i+1})

is a sequence of three consecutive VPs corresponding to IMP margins $I = (i - 1, i, i + 1)$, then the Concavity Rule states:

$$(V_{i+1} - V_i) \leq (V_i - V_{i-1}) \quad (10)$$

That is, consecutive changes in the VP scale must never increase. Computer code to correct for concavity violations depends on the first difference operator `Diff` defined below.

Let $V = [V_1, V_2, \dots, V_n]$ be a vector of length n . Then the first difference of V , is defined by the vector (of length $(n - 1)$)

$$\text{Diff}(V) = [(V_2 - V_1), (V_3 - V_2), \dots, (V_n - V_{n-1})] \quad (11)$$

The pseudo-code to correct concavity violations is presented next.

Continuous Concavity Correction

```

o Get second differences
ddV := [0,0,Diff(round(100*Diff(V)))]
o Get concavity violation test
TestViol := sum(ddV > 0) > 0
o Recursion to correct violations
while TestViol = true
  m := minimum i such that ddV(i) > 0
  if m > 1
    V(m-1) := V(m-1) + 0.01
    Recompute all ddV(i) as above
  end (if loop)
  TestViol := (m is not empty)
end (while loop)

```

Remarks:

- (a) Note the expression $(\text{ddV} > 0)$ is a vector of zeros and ones (a boolean vector). Thus $\text{TestViol} := \text{sum}(\text{ddV} > 0) > 0$, is a boolean variable (0 = false, 1 = true) which determines if a concavity violation has occurred. If there are no such violations, then all the second differences $\text{ddV}(i)$ will be either zero or negative and hence each term $\text{ddV}(i) > 0$ will be zero (*i.e.* false). A concavity violation will therefore show up as a positive $\text{sum}(\text{ddV})$ and then TestViol will equal 1 (*i.e.* true). The two zeros at the start of the vector ddV are necessary in order to avoid infinite recursions.

- (b) The above routine is a recursion which will correct all such detected concavity violations. For example for the case of 8 boards, only one correction is required. However for a 60 board match 24 corrections are required.
- (c) The algorithm above is not unique in correcting for concavity violations. Other correction algorithms are possible, but we have found the above to commonly require fewer iterations to remove all violations, particularly for matches with larger numbers of boards. These other correction algorithms, which include the USBF scales incidentally, will differ from the recommended version above by generally 0.01 of a VP only. So while these differences are tiny and will unlikely change the result of a tournament (let alone a match), consistency across the bridge world will be maintained by everyone sticking to the same algorithm.

A spread-sheet accompanying this report gives the calculated continuous VP scales for several popular board numbers. Programmers can test their codes by comparing with the tables in this spread-sheet.

4 The Discrete VP Scale

Generally, in major national teams events, the organizing body will want to use the new WBF continuous scales. However it has also been anticipated that in some lower level events (*e.g.* congresses and sectionals) the organizing body may prefer to stick to a discrete VP scale, perhaps to avoid the issue of decimal point scales.

This section of the report describes the algorithm to compute discrete VP scales which are commensurate with the new continuous scales.

We shall again describe the winners VP score which will now be an integer value in the range $[0, 20]$. The mapping from IMPs to VPs while still dependent on Eq(1), or more accurately on Eq(8), will no longer be one-to-one. Several integer IMP scores will lead to the same integer VP score. Thus for each VP score V , the mapping will generate a range of integer IMPs I_a to I_b . The end-points of each IMP range, I_a and I_b , are generally referred to as the ‘cusp points’.

The IMP-to-Discrete-VP Algorithm

1. Define the mid-score VPs, V_m in the range $(V_0 + 0.5)$ to $(2V_0 - 0.5)$ in steps of 1-VP. That is,

$$V_m = [10.5, 11.5, 12.5, \dots, 19.5] \quad (12)$$

2. Next use Eq(8) to compute the corresponding IMP scores

$$I_m = \text{floor}\{I_{\text{map}}(V_m; V_0, X)\}; \quad m = (1, 2, \dots, 10)$$

The integer IMPs $I_m = [I_1, I_2, \dots, I_{10}]$ determine a first-pass scale:

IMP-Margin Range	VP score
$[-I_1, I_1]$	10
$[I_1 + 1, I_2]$	11
$[I_2 + 1, I_3]$	12
\vdots	\vdots
$[I_9 + 1, I_{10}]$	19
$[I_{10} + 1, +]$	20

The floor-function of X is defined to be the largest integer not exceeding X (*i.e.* round towards minus infinity). Note that the first or draw range of IMPs is $[-I_1, I_1]$ rather than $[0, I_1]$.

3. The corresponding concavity condition for the discrete scale is:

$$I_2 - I_1 \geq 2I_1 + 1 \quad (13)$$

$$(I_k - I_{k-1}) \geq (I_{k-1} - I_{k-2}); \quad \text{if } k \geq 3 \quad (14)$$

That is, consecutive changes in the IMP range must never decrease. The pseudo-code to identify and correct for concavity violations is presented below.

Discrete Concavity Correction

- Define vector $J = [-(I_1 + 1), I_1, I_2, \dots, I_n]$; (n = 10)
- Get second differences
ddJ := Diff(Diff(J))
- Get violation test
TestViol := sum(ddJ < 0) > 0
- Recursion to correct violations
while TestViol = true
 m := minimum i such that ddJ(i) < 0
 if m > 0
 I(m) := I(m) - 1
 Recompute all ddJ(i) as above
 end (if loop)
 TestViol := (m is not empty)
end (while loop)

The above algorithm to correct for discrete concavity violations is very similar to the corresponding continuous algorithm of Section 3 and can be interpreted in the same way. We have tested this algorithm on all board numbers from 4 to 100 and no concavity violations were detected after the correction recursion was employed.

4.1 Discrete Range Violations

The design of the discrete scale can generate another problem, which fortunately is only a minor one. We call this problem a *range violation* which can be described as follows. Using Eq(8) we can determine an IMP score \hat{I} for a given *integer* VP score \hat{V} say, where \hat{V} can be any of [10, 11, 12, ..., 20]. The \hat{I} values will not in general be integers.

We would like these \hat{I} values to be bracketed by the integer IMP ranges computed by the discrete algorithm described above. That is, we would like, for each i

$$I_{i-1} + 1 \leq \hat{I}_i \leq I_i \tag{15}$$

If this inequality is not satisfied for some i in the range $2 \leq i \leq n$, then we have a range violation. In words, this means that the discrete IMP range does not bracket the continuous scale from which it was derived.

We have tested all board numbers from 4 to 100 and found only three instances of such range violations. These occurred for board numbers: $N = (5, 6, 7)$ and generally the sizes of the violations were quite small. We suggest, for the present, that we can live with these minor variations until such time as a smarter joint concavity-range violation correction algorithm becomes available.

The accompanying spread-sheet contains a page of discrete scales for the same board numbers displaying the continuous scales. All these scales (both continuous and discrete) have been computed using the algorithms described in this report.

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